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Minimum Film Thickness in Elliptical Contacts for Different Regimes of Fluid-Film Lubrication

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SUMMARY

The film-parameter equations are provided for four fluid-film lubrication regimes found in elliptical contacts. These regimes are isoviscous-rigid; viscous-rigid; elastohydrodynamic of low-elastic-modulus materials, or isoviscous-elastic; and elastohydrodynamic, or viscous-elastic. The influence or lack of influence of elastic and viscous effects is the factor that distinguishes these regimes. The film-parameter equations for the respective regimes come from earlier theoretical studies by the authors on elastohydrodynamic and hydrodynamic lubrication of elliptical conjunctions. These equations are restated and the results are presented as a map of the lubrication regimes, with film-thickness contours on a log-log grid of the viscosity and elasticity parameters for five values of the ellipticity parameter. These results present, for the first time, a complete theoretical film-parameter solution for elliptical contacts in the four lubrication regimes.

INTRODUCTION

Two solids, as shown in figure 1, having different radii of curvature in a pair of principal planes (x and y) passing through the contact between the solids, make contact at a single point under the condition of no applied load. When the two solids have a normal load applied to them, the point expands to an ellipse, with a being the semimajor axis and b the semiminor axis. It has been common to refer to elliptical contacts as point contacts. But, since this paper deals with loaded contacts, the term elliptical contact is used. For the special case where $r_{Ay} = r_{Ax}$ and $r_{By} = r_{Bx}$, the resulting contact is a circle rather than an ellipse. Furthermore, as either r_{Ay} or r_{By} becomes much larger than the remaining radii of curvature, the elliptical contact approaches a rectangular contact. A rectangular contact exists when side leakage of the fluid can be ignored in comparison with the fluid flow in the rolling direction. Therefore, this paper is concerned with conjunctions between solids with contact regions ranging from circular to rectangular.

The types of lubrication within the contact shown in figure 1 are influenced by two major physical effects: the elastic deformation of the solids under an applied load, and the increase in fluid viscosity with pressure. Therefore it is possible to have four regimes of fluid-film lubrication, depending on the magnitude of these effects and on

their relative importance. These four regimes are defined as

(1) Isoviscous-rigid: In this regime the elastic deformation of the surfaces is an insignificant part of the fluid film separating them, and the maximum pressure in the contact is too low to significantly increase fluid viscosity. This form of lubrication is typically used in circular-arc thrust bearing pads; industrial coating processes in which paint, emulsion, or protective coatings are applied to sheet or film materials passing between rollers; and very lightly loaded cylindrical roller bearings.

(2) Viscous-rigid: In this regime the pressure within the contact is high enough so that there is a significant increase in the fluid viscosity within the contact. The deformation of the surfaces is an insignificant part of the fluid-film thickness. This form of lubrication is used on roller end-guide flanges, in contacts in moderately loaded cylindrical and tapered rollers, and between some piston rings and cylinder liners.

(3) Isoviscous-elastic: In this regime the elastic deformation of the solids is a significant part of the thickness of the fluid film separating them. The pressure within the contact is low and close to that obtained under dry frictionless conditions. This form of lubrication is used in seals, human joints, tires, and elastomeric-material machine elements. These applications are typified by solid materials with low elastic modulus.

(4) Viscous-elastic: In this regime the elastic deformation of the solids is also a significant part of the thickness of the fluid film separating them, and the pressure within the contact is high enough to cause a significant increase in the fluid viscosity within the contact. This form of lubrication is used in ball and roller bearings, gears, and cams.

Several authors (refs. 1 to 6) have contributed solutions for the film thickness in the four lubrication regimes, but their results have been confined largely to rectangular contacts. The essential difference between these contributions is the way the parameters were made dimensionless. In the present paper the film thickness is defined for the four fluid-film lubrication regimes just described for conjunctions ranging from circular to rectangular. The film-thickness equations for the respective regimes come from theoretical studies reported in references 7 to 9 on elastohydrodynamic and hydrodynamic lubrication of elliptical conjunctions. The results presented herein are valid for isothermal fully flooded conjunctions. In addition to the film-thickness equations for the respective regimes, a map is presented of the lubrication regimes, with film-thickness contours on a log-log grid of the viscosity and elasticity parameters for five values of the ellipticity parameter.

SYMBOLS

a	semimajor axis of contact ellipse
b	semiminor axis of contact ellipse
E	modulus of elasticity
E'	$\dfrac{2}{\left(\dfrac{1-\nu_A^2}{E_A}+\dfrac{1-\nu_B^2}{E_B}\right)}$
F	normal applied load
G	dimensionless material parameter, $E'/p_{iv,as}$
g_E	dimensionless elasticity parameter, $W^{8/3}/U^2$
g_V	dimensionless viscosity parameter, GW^3/U^2
H	dimensionless film thickness, h/R_x
$\hat{H}$	dimensionless film parameter, $H\left(\dfrac{W}{U}\right)^2 = \dfrac{F^2h}{u^2\eta_0^2R_x^3}$
h	film thickness
k	ellipticity parameter, a/b
$p_{iv,as}$	asymptotic isoviscous pressure
p_0	maximum Hertzian pressure
q_f	reduced pressure parameter
R	effective radius
r	radius of curvature
U	dimensionless speed parameter, $u\eta_0/E'R_x$
u	mean surface velocity in x-direction, $\frac{1}{2}(u_A + u_B)$
W	dimensionless load parameter, $F/E'R_x^2$
x, y	Cartesian coordinates
α	pressure-viscosity coefficient

β	R_y/R_x
η_0	fluid viscosity at 1-atmosphere pressure
ν	Poisson's ratio
Φ	$\left(1+\frac{2}{3\beta}\right)^{-1}$

Subscripts:

A	solid A
B	solid B
c	central
E	elastic
I	isoviscous
min	minimum
R	rigid
V	viscous
x,y	in x and y directions, respectively

DIMENSIONLESS GROUPING

The dimensionless groupings used herein were also used by the authors in earlier publications (refs. 7 to 9) and are

Dimensionless film thickness:

$$H = \frac{h}{R_x} \tag{1}$$

where

$$\frac{1}{R_x} = \frac{1}{r_{Ax}} + \frac{1}{r_{Bx}} \tag{2}$$

Dimensionless speed parameter:

$$U = \frac{\eta_0 u}{E' R_x} \tag{3}$$

where

$$u = \frac{u_A + u_B}{2} \tag{4}$$

$$E' = \frac{2}{\left(\frac{1 - \nu_A^2}{E_A} + \frac{1 - \nu_B^2}{E_B} \right)} \tag{5}$$

Dimensionless load parameter:

$$W = \frac{F}{E' R_x^2} \tag{6}$$

Dimensionless material parameter:

$$G = \frac{E'}{p_{iv,\,as}} \approx \alpha E' \tag{7}$$

Ellipticity parameter:

$$k = \frac{a}{b} \tag{8}$$

The conventional method of calculating the ellipticity parameter is to find a solution to a transcendental equation that relates the ellipticity parameter and the elliptic integrals of the first and second kind to the geometry of the contacting solids. This is usually accomplished by some iterative numerical procedure. The following simple expression for the ellipticity parameter, which eliminates the necessity for that procedure, is outlined in reference 10:

$$k = 1.03 \left(\frac{R_y}{R_x} \right)^{0.64} \tag{9}$$

where

$$\frac{1}{R_y} = \frac{1}{r_{Ay}} + \frac{1}{r_{By}} \tag{10}$$

The approximate solution of the ellipticity parameter as obtained from equation (9) is within 3 percent of the exact solution for ellipticity parameters between 1 and 10.

The dimensionless group $\{H, U, W, G, k\}$ has been a useful tool in understanding the results of references 7 to 9. Several authors (refs. 1 to 6) have noted that this dimensionless group can be reduced by one parameter - without any loss of generality - by using dimensional analysis. The film-thickness contours for the four fluid-film lubrication regimes are best represented graphically by the fewest parameters.

Johnson (ref. 5) points out that the behavior distinguishing the four lubrication regimes can be characterized by three quantities, each having the dimensions of pressure, namely:

(1) The reduced pressure parameter q_f, a measure of the fluid pressure generated by an isoviscous lubricant when elastic deformation is neglected

(2) The inverse pressure-viscosity coefficient $1/\alpha$, a measure of the change of viscosity with pressure

(3) The maximum Hertzian pressure p_0, the maximum pressure of a dry elastic contact

Although Johnson's paper (ref. 5) does not consider elliptical contacts, it does state what the nondimensional parameters would be, namely:

Dimensionless film parameter:

$$\hat{H} = H\left(\frac{W}{U}\right)^2 \tag{11}$$

Dimensionless viscosity parameter:

$$g_V = \alpha q_f = \frac{GW^3}{U^2} \tag{12}$$

Dimensionless elasticity parameter:

$$g_E = \frac{1}{\pi}\left(\frac{3}{2}\right)^{1/3}\left(\frac{q_f}{p_0}\right) = \frac{W^{8/3}}{U^2} \tag{13}$$

The ellipticity parameter k remains as discussed in equations (8) and (9). Therefore the reduced dimensionless group is $\{\hat{H}, g_V, g_E, k\}$.

Isoviscous-Rigid Regime

The influence of geometry on the isothermal hydrodynamic film separating two rigid solids was investigated in reference 9 for fully flooded isoviscous conditions. The effect of geometry on the film thickness was determined by varying the radius ratio R_y/R_x from 1 (a circular configuration) to 36 (a configuration approaching a rectangular contact). The film thickness was varied over two orders of magnitude for steel solids separated by a paraffinic mineral oil. It was found that the minimum film thickness had the same speed, viscosity, and load dependence as the classical Kapitza (ref. 11) solution. However, incorporating the Reynolds cavitation boundary condition (i.e., $\partial p/\partial n = 0$ and $p = 0$ at the cavitation boundary, where n represents the normal coordinate to the cavitation boundary) resulted in an additional geometry effect. Therefore, from reference 9 the dimensionless minimum (or central) film parameter for the isoviscous-rigid lubrication regime can be written as

$$\left\{\hat{H}_{min}\right\}_{IR} = \left\{\hat{H}_c\right\}_{IR} = 128\beta\Phi^2\left[0.13\tan^{-1}\left(\frac{\beta}{2}\right) + 1.68\right]^2 \tag{14}$$

where

$$\beta = \frac{R_y}{R_x} \approx \left(\frac{k}{1.03}\right)^{1/0.64} \tag{15}$$

$$\Phi = \left(1 + \frac{2}{3\beta}\right)^{-1} \tag{16}$$

In equation (14) the dimensionless film parameter is strictly a function of the geometry of the contact R_y/R_x.

Viscous-Rigid Regime

From Blok (ref. 12) we can write the minimum film thickness for the viscous-rigid lubrication regime in a rectangular contact as

$$h_{min} = h_c = 1.66\left(\alpha^2\eta_0^2u^2R_x\right)^{1/3} \tag{17}$$

By using the elliptical effect from reference 7, equation (17) can be rewritten as

$$h_{min} = h_c = 1.66\left(\alpha^2\eta_0^2u^2R_x\right)^{1/3}\left(1 - e^{-0.68k}\right) \tag{18}$$

Note the absence of an applied-load term in equation (18). When expressed in terms of the dimensionless parameters of equations (11) and (12), this can be written as

$$\{\hat{H}_{min}\}_{VR} = \{\hat{H}_c\}_{VR} = 1.66\, g_V^{2/3}(1 - e^{-0.68k}) \tag{19}$$

Note the absence of the dimensionless elasticity parameter g_E in equation (19).

Isoviscous-Elastic Regime

The influence of the ellipticity parameter k and the dimensionless speed U, load W, and material G parameters on the minimum and central film thicknesses was investigated theoretically for the isoviscous-elastic regime in reference 8. The ellipticity parameter was varied from 1 (a circular configuration) to 12 (a configuration approaching a rectangular contact). The dimensionless speed and load parameters were each varied by one order of magnitude. Seventeen cases were considered in obtaining the dimensionless minimum-film-thickness equation

$$H_{min} = 7.43\, U^{0.65} W^{-0.21}(1 - 0.85\, e^{-0.31k}) \tag{20}$$

From equations (11) and (13) the general form of the dimensionless minimum-film-parameter equation for the isoviscous-elastic lubrication regime can be expressed as

$$\hat{H}_{min} = A g_E^c (1 - 0.85\, e^{-0.31k}) \tag{21}$$

where A and c are constants to be determined. From equations (11) and (13), equation (21) becomes

$$H_{min} = A U^{2-2c} W^{(8/3c)-2}(1 - 0.85\, e^{-0.31k}) \tag{22}$$

Comparing equation (20) with equation (22) gives $c = 0.67$. Substituting this into equation (21) while solving for A gives

$$A = \frac{\hat{H}_{min}}{g_E^{0.67}(1 - 0.85\, e^{-0.31k})} \tag{23}$$

The arithmetic mean for A based upon the 17 cases considered in reference 8 is 8.70, with a standard deviation of ± 0.05. Therefore the dimensionless minimum film parameter for the isoviscous-elastic lubrication regime can be written as

$$\{\hat{H}_{min}\}_{IE} = 8.70\, g_E^{0.67}(1 - 0.85\, e^{-0.31k}) \tag{24}$$

With a similar approach the dimensionless central film parameter for the isoviscous-elastic lubrication regime can be written as

$$\{\hat{H}_c\}_{IE} = 11.15\, g_E^{0.67}(1 - 0.72\, e^{-0.28k}) \tag{25}$$

Viscous-Elastic Regime

In reference 7 the influence of the ellipticity parameter k and the dimensionless speed U, load W, and material G parameters on the minimum and central film thicknesses was investigated theoretically for the viscous-elastic regime. The ellipticity parameter was varied from 1 to 8, the dimensionless speed parameter was varied over nearly two orders of magnitude, and the dimensionless load parameter was varied over one order of magnitude. Conditions corresponding to the use of solid materials of bronze, steel, and silicon nitride and lubricants of paraffinic and naphthenic oils were considered in obtaining the exponent on the dimensionless materials parameter. Thirty-four cases were used in obtaining the following dimensionless minimum-film-thickness formula

$$H_{min} = 3.63\, U^{0.68} G^{0.49} W^{-0.073}(1 - e^{-0.68k}) \tag{26}$$

The general form of the dimensionless film-parameter equation for the viscous-elastic lubrication regime can be written as

$$\hat{H}_{min} = B g_V^d g_E^f (1 - e^{-0.68k}) \tag{27}$$

where B, d, and f are constants to be determined. From equations (11), (12), and (13) equation (27) can be rewritten as

$$H_{min} = B G^d U^{2-2d-2f} W^{-2+3d+(8f/3)}(1 - e^{-0.68k}) \tag{28}$$

Comparing equation (26) with equation (28) gives $d = 0.49$ and $f = 0.17$. Substituting these values into equation (27) and solving for B give

$$B = \frac{\hat{H}_{min}}{g_V^{0.49} g_E^{0.17}(1 - e^{-0.68k})} \tag{29}$$

For the 34 cases used in reference 7 in obtaining equation (26), the arithmetic mean for B was 3.42, with a standard derivation of ±0.03. Therefore, the dimensionless minimum film parameter for the viscous-elastic lubrication regime can be written as

$$\{\hat{H}_{min}\}_{VE} = 3.42\, g_V^{0.49} g_E^{0.17}(1 - e^{-0.68k}) \tag{30}$$

An interesting observation to make in comparing equations (19), (24), and (30) is that the sum of the exponents on g_V and g_E is close to the value of 2/3 required for complete dimensional representation of these three lubrication regimes: viscous-rigid, isoviscous-elastic, and viscous-elastic.

With an approach like that just described, the dimensionless central film parameter for the viscous-elastic lubrication regime can be written as

$$\{\hat{H}_c\}_{VE} = 3.61\, g_V^{0.53} g_E^{0.13}(1 - 0.61\, e^{-0.73k}) \tag{31}$$

PROCEDURE FOR MAPPING LUBRICATION REGIMES

Having expressed the dimensionless minimum film parameters for the four lubrication regimes in equations (14), (19), (24), and (30), we used these equations to develop a map of the lubrication regimes in the form of dimensionless film-parameter contours. These maps are shown in figures 2 to 6 on a log-log grid of the dimensionless viscosity and elasticity parameters for ellipticity parameters of 1, 2, 3, 4, and 6, respectively. The procedure used to obtain these figures was as follows:

(1) For a given ellipticity parameter k, $\{\hat{H}_{min}\}_{IR}$ was calculated from equation (14).

(2) For a value of $\hat{H}_{min} \geq \{H_{min}\}_{IR}$ and the ellipticity parameter k chosen in step 1, the dimensionless viscosity parameter g_V was calculated from equation (19) as

$$g_V = \left[\frac{\hat{H}_{min}}{1.66(1 - e^{-0.68k})}\right]^{3/2} \tag{32}$$

This established the dimensionless film-parameter contour $\hat{H}_{min}$ in the viscous-rigid regime.

(3) For the values of k selected in step 1, $\hat{H}_{min}$ selected in step 2, and g_V obtained from equation (32), the dimensionless elasticity parameter was calculated from the following equation, which was obtained from equation (30):

$$g_E = \left[\frac{\hat{H}_{min}}{3.42\, g_V^{0.49}(1 - e^{-0.68k})} \right]^{1/0.17} \tag{33}$$

This established the boundary between the viscous-rigid and viscous-elastic regimes.

(4) For the values of k and $\hat{H}_{min}$ chosen in steps 1 and 2, the dimensionless elasticity parameter was calculated from the following equation obtained by rearrangement of equation (24):

$$g_E = \left[\frac{\hat{H}_{min}}{8.70(1 - 0.85\, e^{-0.31k})} \right]^{1/0.67} \tag{34}$$

This established the dimensionless film-parameter contour $\hat{H}_{min}$ in the isoviscous-elastic lubrication regime.

(5) For the values of k and $\hat{H}_{min}$ selected in steps 1 and 2 and the value of g_E obtained from equation (34), g_V was calculated from the following equation:

$$g_V = \left[\frac{\hat{H}_{min}}{3.42\, g_E^{0.17}(1 - e^{-0.68k})} \right]^{1/0.49} \tag{35}$$

This established the isoviscous-elastic and viscous-elastic boundaries for the particular values of k and $\hat{H}_{min}$ chosen in steps 1 and 2.

(6) At this point, for particular values k and $\hat{H}_{min}$, the contours were drawn through the viscous-rigid, viscous-elastic, and isoviscous-elastic regimes. A new $\hat{H}_{min}$ value was then selected, and the contour was constructed by returning to step 2. This procedure was continued until an adequate number of contours had been generated. A similar procedure was followed for the range of k values considered.

DISCUSSION OF CONTOUR PLOTS

The maps of the lubrication regimes shown in figures 2 to 6 were generated by following the procedure outlined in the previous section. The dimensionless-film-parameter $\hat{H}_{min}$ contours were plotted on a log-log grid of the dimensionless viscosity parameter g_V and the dimensionless elasticity parameter g_E for ellipticity parameters of 1, 2, 3, 4, and 6, respectively. The four lubrication regimes are clearly shown in these figures. The smallest film-parameter contour represents the value obtained from equation (14) and is the isoviscous-rigid boundary. The value of

this dimensionless film parameter for the isoviscous-rigid boundary increases as the ellipticity parameter k increases.

By using figures 2 to 6 for given values of the ellipticity parameter k, the viscosity parameter g_V, and the elasticity parameter g_E, the lubrication regime can be ascertained and the approximate value of the dimensionless film parameter $\hat{H}_{min}$ determined. When the lubrication regime is known, a more accurate value of $\hat{H}_{min}$ can be obtained by using the appropriate dimensionless-film-parameter equation.

Figures 7 to 11 were obtained by using the dimensionless-film-parameter equations for the four lubrication regimes and a procedure like that used in obtaining figures 2 to 6. The difference between the two groups of figures is that the ordinate in figures 2 to 6 is the viscosity parameter g_V and the contours are values of the dimensionless film parameter $\hat{H}_{min}$ but the ordinate in figures 7 to 11 is $\hat{H}_{min}$ and the contours are values of g_E. In figures 7 to 11 the boundaries of the isoviscous-rigid and isoviscous-elastic regimes are shown, as are the viscous-rigid and viscous-elastic regimes.

Figure 12 gives a three-dimensional view of the surfaces of three constant dimensionless film parameters ($\hat{H}_{min}$ = 500, 2000, and 6000). The coordinates in this figure are the dimensionless elasticity parameter g_E, the dimensionless viscosity parameter g_V, and the ellipticity parameter k. The four fluid-film lubrication regimes are clearly shown. In this figure, we attempt to show how the parameters g_V, g_E, and k influence the dimensionless film parameter $\hat{H}_{min}$ as well as to define the fluid-film lubrication regimes.

CONCLUSIONS

The dimensionless minimum-film-parameter $\hat{H}_{min}$ equations for the four lubrication regimes found in elliptical contacts are

Isoviscous-rigid regime:

$$\left\{\hat{H}_{min}\right\}_{IR} = 128\beta\Phi^2\left[0.13\tan^{-1}\left(\frac{\beta}{2}\right) + 1.68\right]^2$$

where

$$\beta = \frac{R_y}{R_x} \approx \left(\frac{k}{1.03}\right)^{1/0.64}$$

$$\Phi = \left(1 + \frac{2}{3\beta}\right)^{-1}$$

Viscous-rigid regime:

$$\{\hat{H}_{min}\}_{VR} = 1.66\, g_V^{2/3}(1 - e^{-0.68k})$$

Isoviscous-elastic regime:

$$\{\hat{H}_{min}\}_{IE} = 8.70\, g_E^{0.67}(1 - 0.85\, e^{-0.31k})$$

Viscous-elastic regime:

$$\{\hat{H}_{min}\}_{VE} = 3.45\, g_V^{0.49} g_E^{0.17}(1 - e^{-0.68k})$$

where R_x and R_y are the effective radii in the x and y directions, k is the ellipticity parameter, g_V is the dimensionless viscosity parameter, and g_E is the dimensionless elasticity parameter. The relative importance of the elastic and viscous effects is the factor that distinguishes these regimes.

In addition these equations have been used to map the lubrication regimes by plotting film-thickness contours on a log-log grid of the viscosity and elasticity parameters for five values of the ellipticity parameter. These results present, for the first time, a complete theoretical film-parameter solution for elliptical contacts in the four lubrication regimes. The results are expected to be most useful in the solution of many practical lubrication problems.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 29, 1978,
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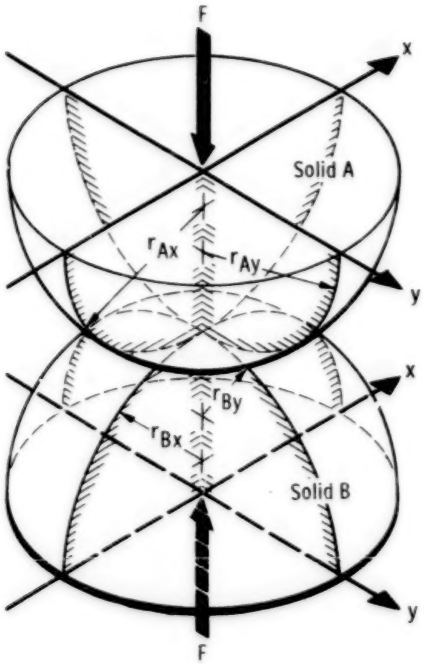

Figure 1. - Geometry of contacting elastic solids.

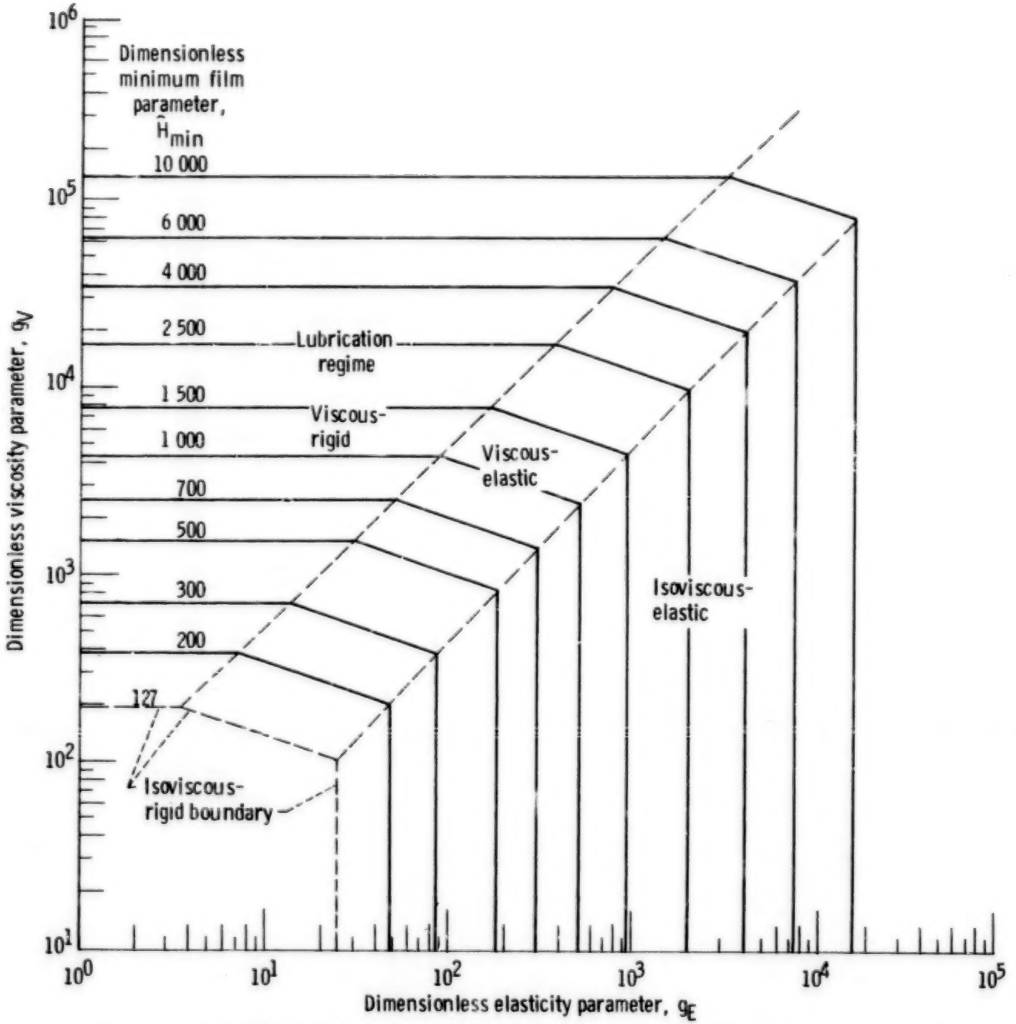

Figure 2. - Map of lubrication regimes with dimensionless-film-parameter contours on log-log grid of dimensionless viscosity and elasticity parameters for ellipticity parameter of 1 (k = 1).

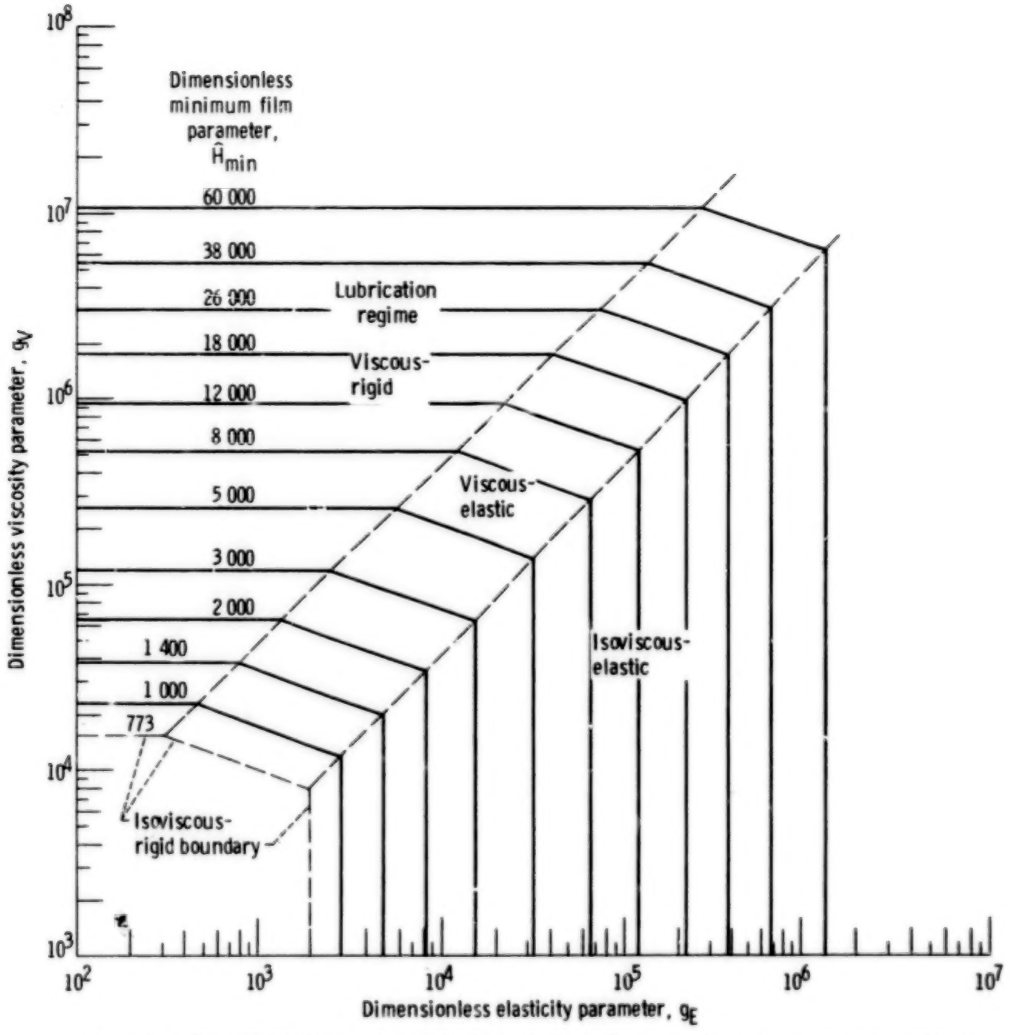

Figure 3. - Map of lubrication regimes with dimensionless-film-parameter contours on log-log grid of dimensionless viscosity and elasticity parameters for ellipticity parameter of 2 (k = 2).

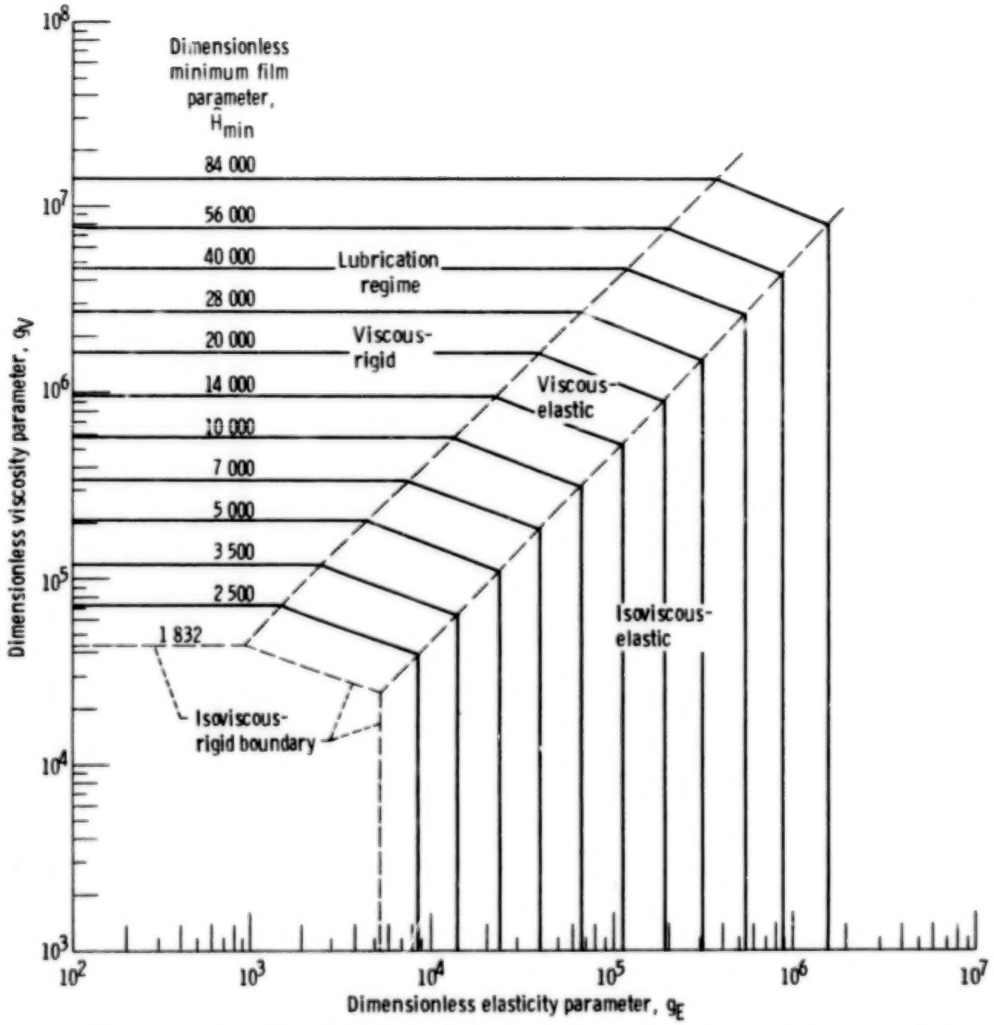

Figure 4. - Map of lubrication regimes with dimensionless-film-parameter contours on log-log grid of dimensionless viscosity and elasticity parameters for ellipticity parameter of 3 (k = 3).

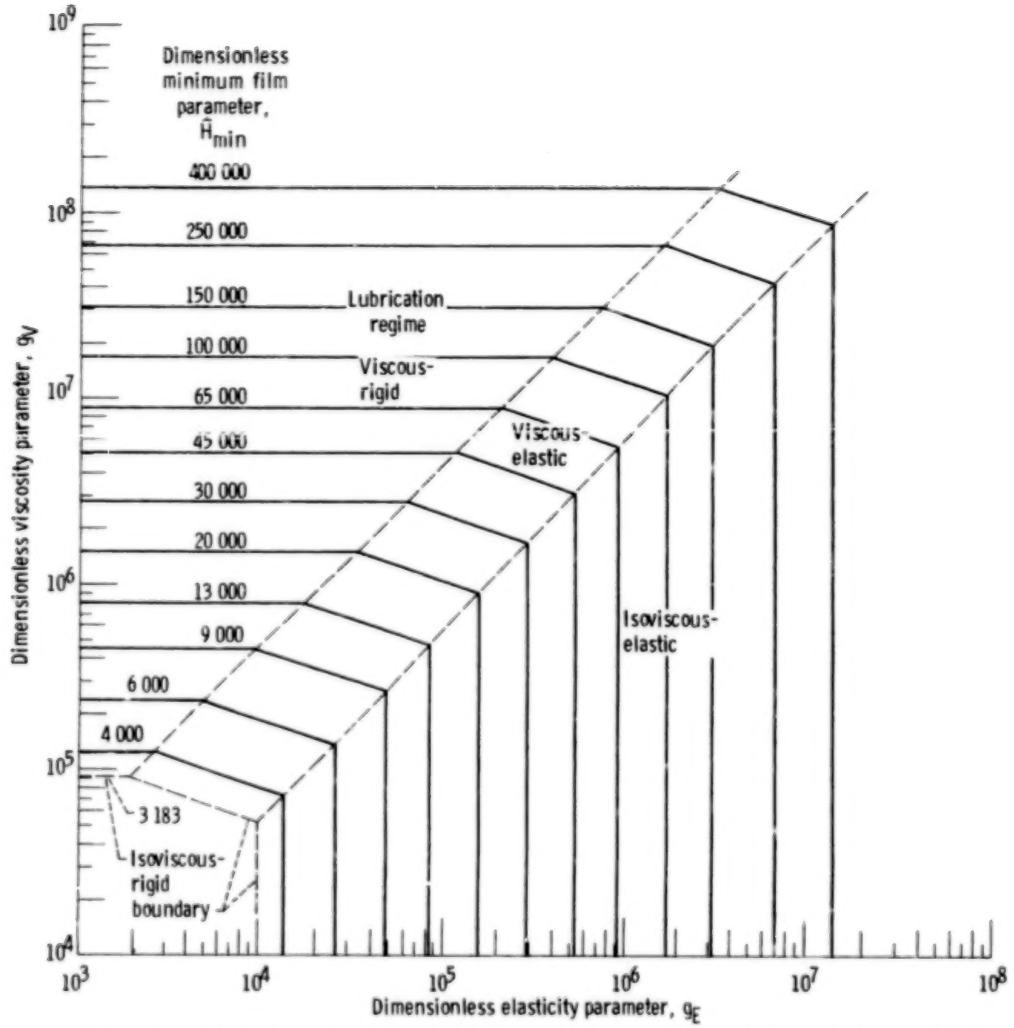

Figure 5. - Map of lubrication regimes with dimensionless-film-parameter contours on log-log grid of dimensionless viscosity and elasticity parameters for ellipticity parameter of 4 (k = 4).

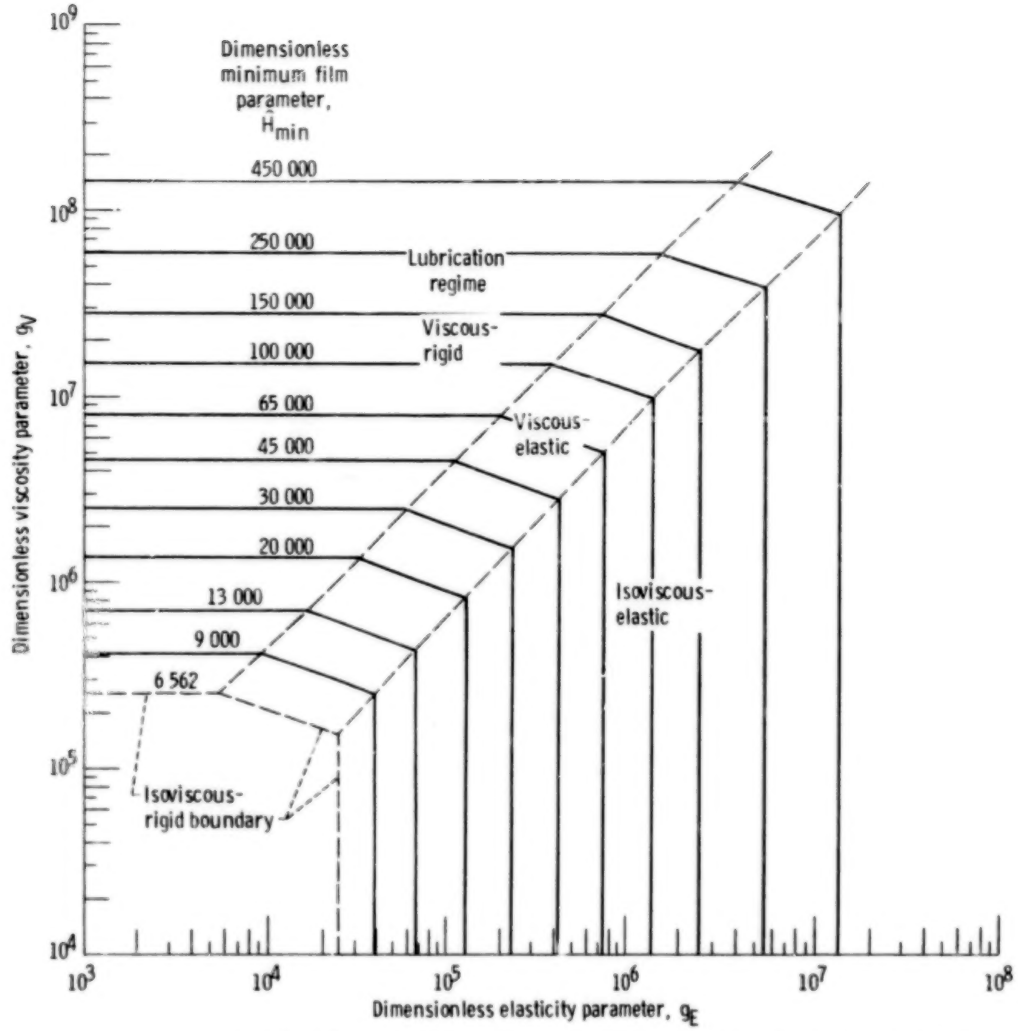

Figure 6. - Map of lubrication regimes with dimensionless-film-parameter contours on log-log grid of dimensionless viscosity and elasticity parameters for ellipticity parameter of 6 (k = 6).

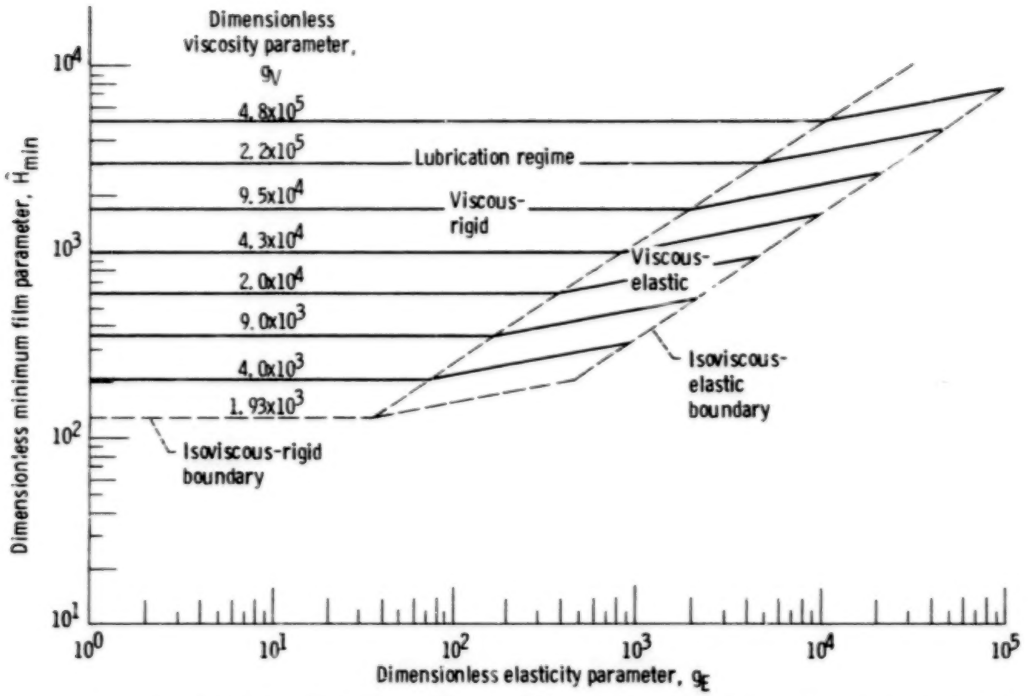

Figure 7. - Map of lubrication regimes with dimensionless-viscosity-parameter contours on log-log grid of dimensionless film and elasticity parameters for ellipticity parameter of 1 (k = 1).

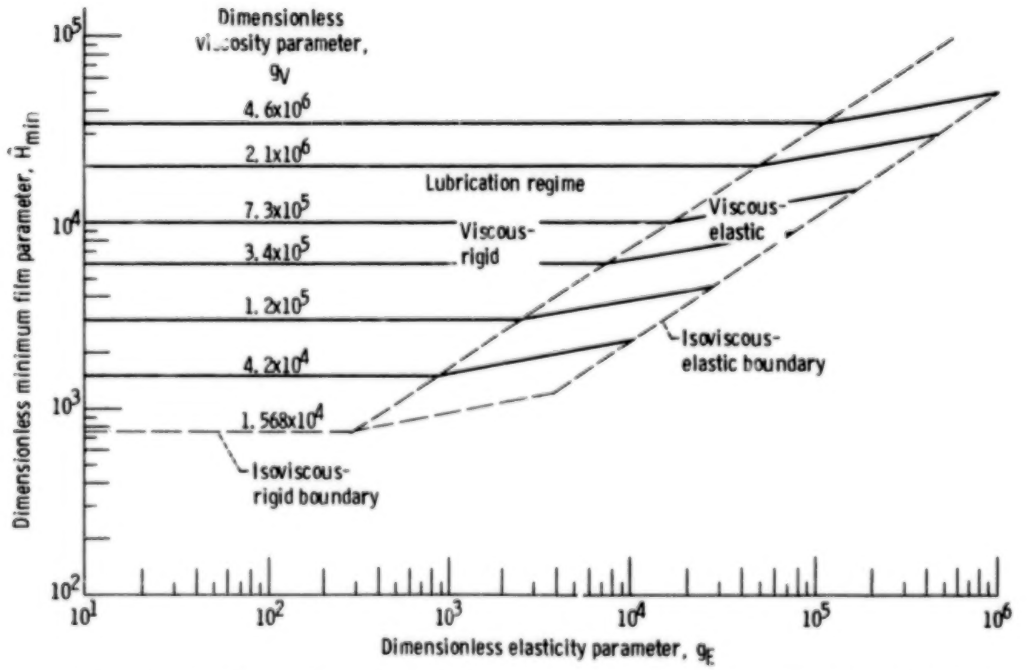

Figure 8. - Map of lubrication regimes with dimensionless-viscosity-parameter contours on log-log grid of dimensionless film and elasticity parameters for ellipticity parameter of 2 (k = 2).

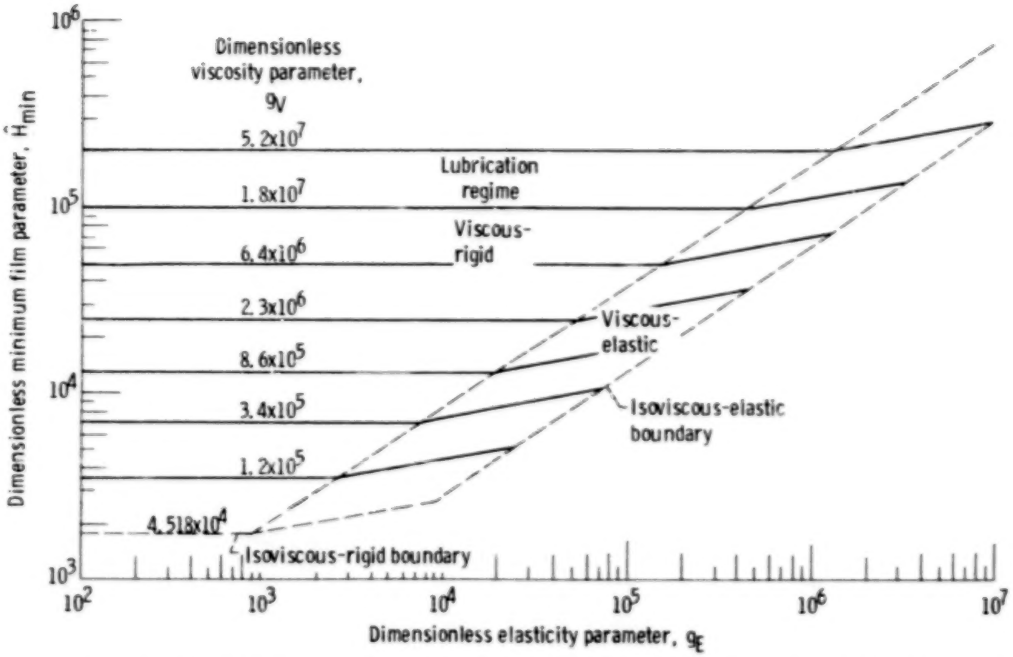

Figure 9. - Map of lubrication regimes with dimensionless-viscosity-parameter contours on log-log grid of dimensionless film and elasticity parameters for ellipticity parameter of 3 (k = 3).

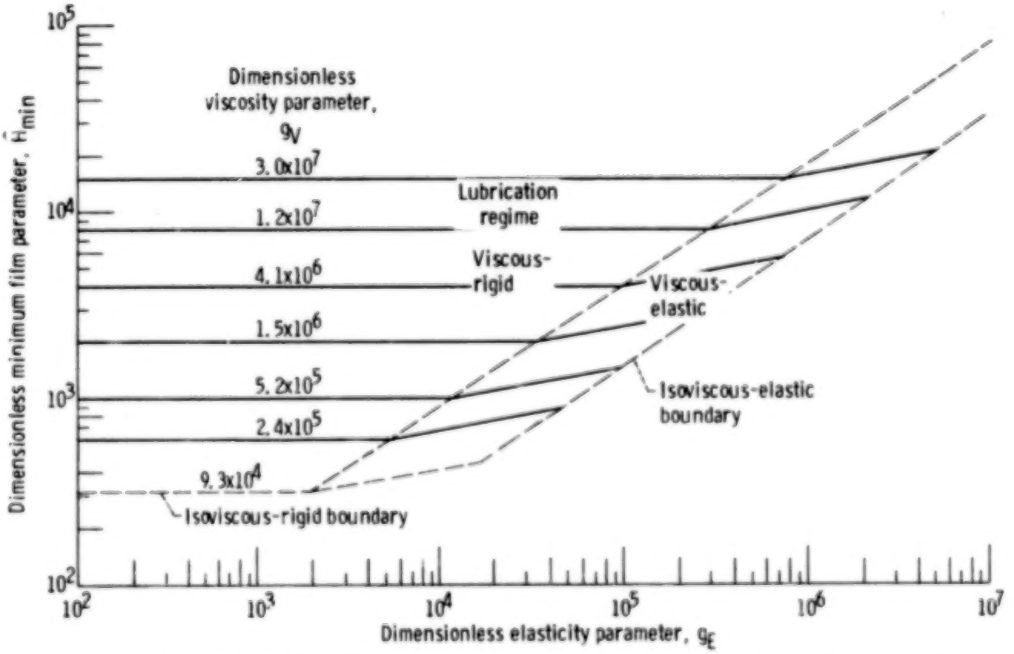

Figure 10. - Map of lubrication regimes with dimensionless-viscosity-parameter contours on log-log grid of dimensionless film and elasticity parameters for ellipticity parameter of 4 (k = 4).

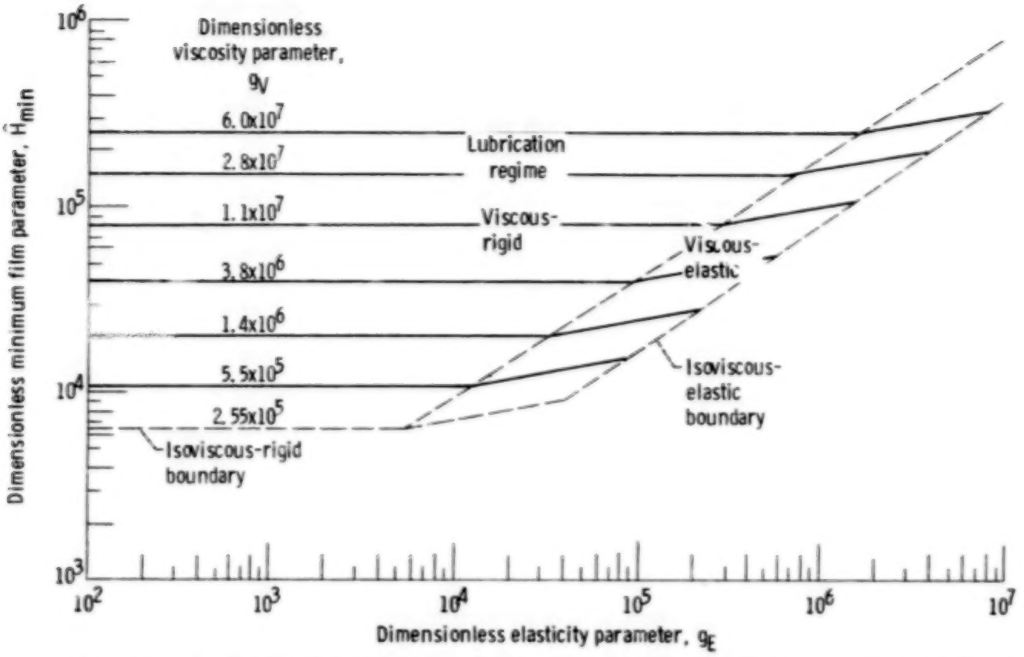

Figure 11. - Map of lubrication regimes with dimensionless-viscosity-parameter contours on log-log grid of dimensionless film and elasticity parameters for ellipticity parameter of 6 (k = 6).

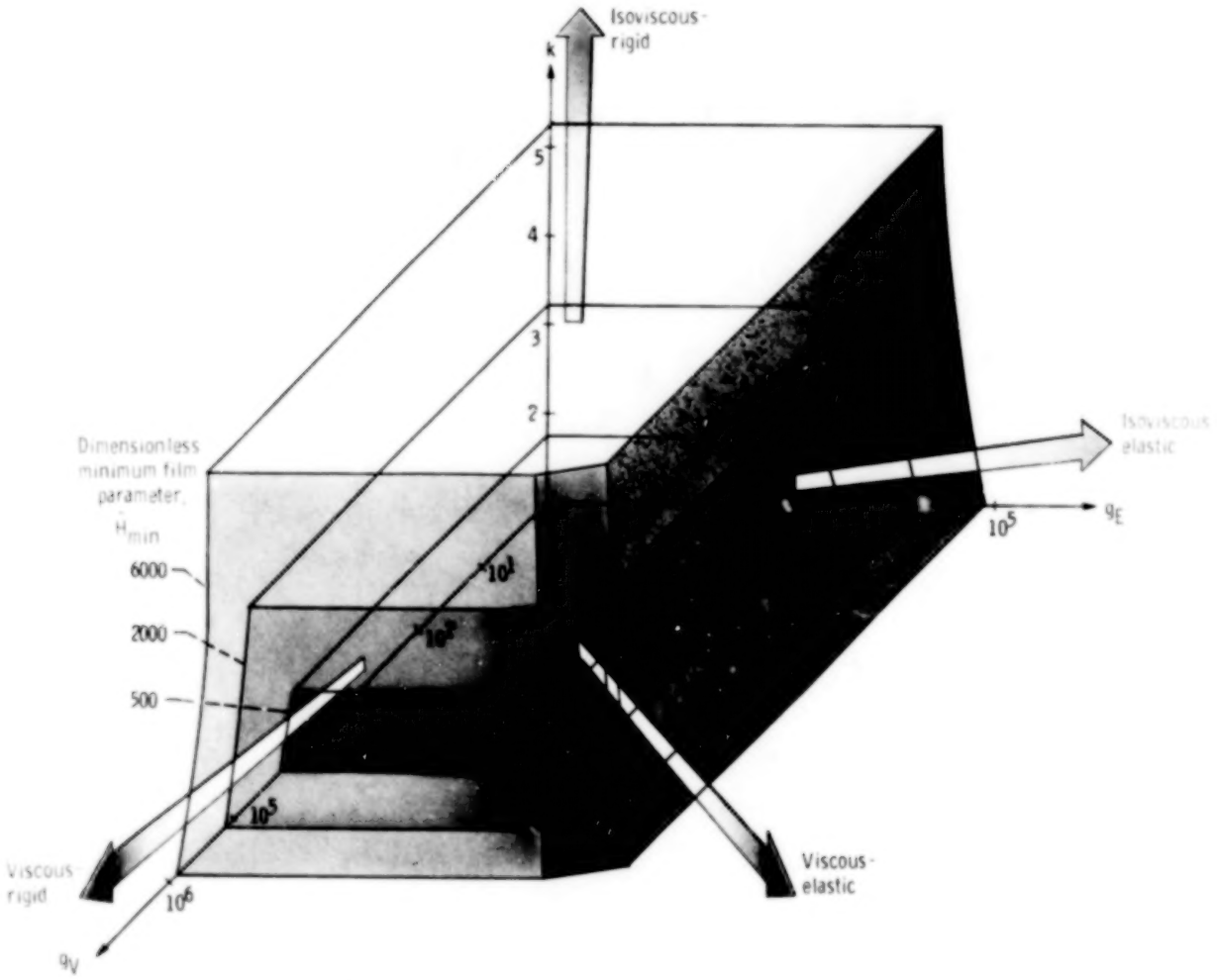

Figure 12. - Surfaces of constant dimensionless film parameter.

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15. Supplementary Notes

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16. Abstract

The film-parameter equations are provided for four fluid-film lubrication regimes found in elliptical contacts. These regimes are isoviscous-rigid; viscous-rigid; elastohydrodynamic of low-elastic-modulus materials, or isoviscous-elastic; and elastohydrodynamic, or viscous-elastic. The influence or lack of influence of elastic and viscous effects is the factor that distinguishes these regimes. The film-parameter equations for the respective regimes come from earlier theoretical studies by the authors on elastohydrodynamic and hydrodynamic lubrication of elliptical conjunctions. These equations are restated and the results are presented as a map of the lubrication regimes, with film-thickness contours on a log-log grid of the viscosity and elasticity parameters for five values of the ellipticity parameter. These results present, for the first time, a complete theoretical film-parameter solution for elliptical contacts in the four lubrication regimes.

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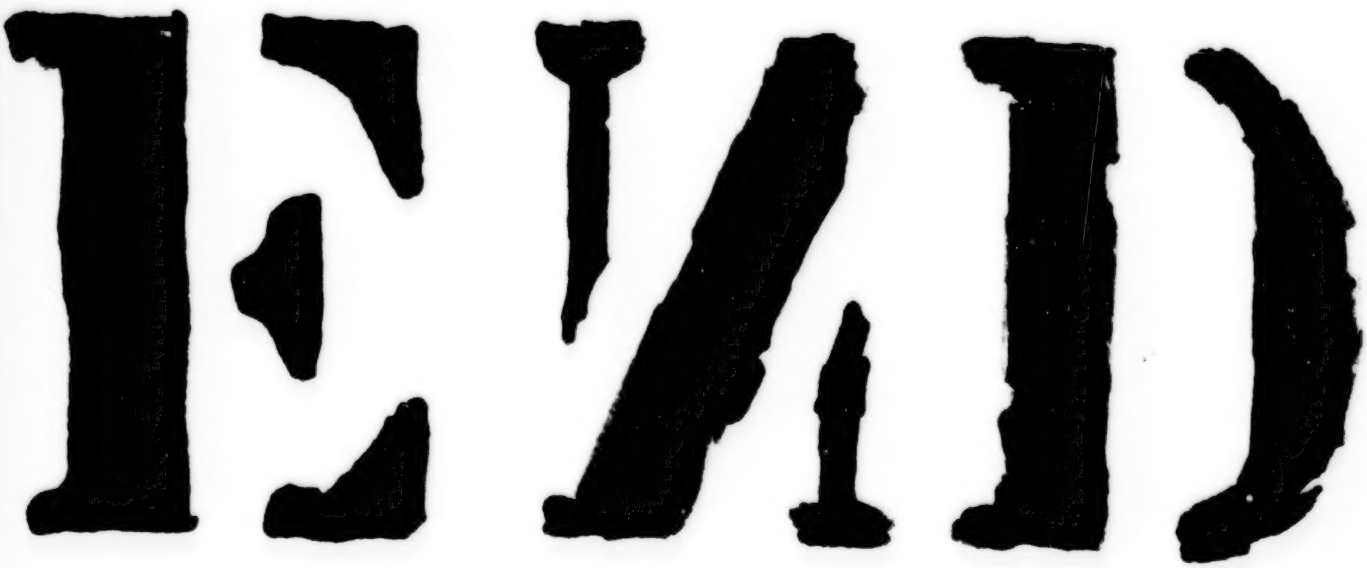
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